

## TAXATION OF CAPITAL GAINS AND THE BEHAVIOR OF STOCK PRICES OVER THE DIVIDEND CYCLE

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### 1. Introduction

This paper seeks to challenge prevailing perceptions concerning the effect of capital gains tax upon the behavior of stock prices over the dividend cycle. According to the traditional view, a share value should systematically appreciate over the dividend cycle at shareholders' market-determined opportunity rate of interest and drop when the stock goes ex-dividend by the amount distributed. This view, which found support in early studies (e.g., [2], [4], [6]), has been criticized more recently in a much quoted study by Elton and Gruber [5]. Focusing their attention on ex-dividend price behavior, these authors argue that the extent of the drop in price upon distribution should be determined by shareholders' tax bracket (p. 69):

A stockholder selling stock before a stock goes ex-dividend loses the right to the already declared dividend. If he sells the stock on the ex-dividend day he retains the dividend but should expect to sell it at a lower price (because of this dividend retention). In a rational market the fall in price on the ex-dividend day should reflect the value of dividends vis-à-vis capital gains to the marginal stockholders. Since dividends and capital gains are taxable at different rates, the relative tax rate on these two types of income affect the decision.

Noting that at any given bracket capital gains are taxed at a lower rate than dividends, the authors seek to support this thesis with evidence showing that the ex-dividend price drop is systematically smaller than the amount of dividend per share.

Theoretical results of this study are consistent with Elton and Gruber's empirical findings regarding share price behavior on ex-dividend dates, but present a challenge to their interpre-

tation of those findings. It is argued here that although shareholders' marginal tax bracket should affect the ex-dividend price drop, the mechanism involved is more complex than previously perceived. Specifically, it is shown that in focusing on the effect of capital gains tax on the price asked by mid-cycle sellers, Elton and Gruber overlook a parallel effect of capital loss credit on the bid price offered by mid-cycle buyers. These effects are shown to create a bid-ask price dichotomy with a corresponding dual price drop on ex-dividend dates.<sup>1</sup>

Findings of this paper have further implications for the firm's optimal scheduling of dividends and the investor's timing of stock trading, as well as for government policy in designing a tax system. With respect to the firm's policy, results show that the timing and frequency of distribution affect the return earned by investors trading stock between dividend dates. Proper timing and higher frequency of distribution should reduce the tax penalty imposed on mid-cycle traders. As applied to individual investors, results indicate that this penalty is totally avoided when trading is limited to ex-dividend dates. Regarding government policy, results point to the existence of an undesirable side effect of capital gains tax which could be reduced and possibly avoided by maintaining parity between the rates of capital gains tax and capital loss credit.

Finally, results presented have implications for the interpretation of the observed bid-ask spread in stock prices. These results draw attention to a source of price spread ignored by previous writers—the tax treatment of capital gains and losses realized in stock trading (e.g., [1], [3]). The analysis offered indicates that the contribution of tax effects to the overall spread should be determined by the timing and frequency of dividends, the tax and credit rates on capital gains and losses, and investors' opportunity rate of return.

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Results are presented according to the following plan:

- (i) The ask price of a non-growth stock allowing for the effect of capital gains tax appreciates between dividends at an *accelerating* rate rather than a fixed exponential rate. That rate is increasing with the rate of capital gains tax and investors' opportunity rate of return (Section 2).
- (ii) The bid price allowing for the effect of capital loss credit behaves in a fashion similar to that of the ask price. The former appreciates at a slower pace than the latter to the extent that loss credit is at a lower rate than gains tax. The spread between the two price paths increases with the difference between these rates and with investors' opportunity rate of return; it vanishes only on ex-dividend dates (Section 3).
- (iii) Market equilibrium in the presence of capital gains tax dictates domination by ex-dividend trading (Section 4).
- (iv) Bid and ask price paths allowing for the effect of capital gains tax are affected differently by the timing of dividends within the tax year (Section 5).
- (v) The value of a stock may be influenced by the timing of dividends due to the effect of the bid-ask spread on the tax penalty sustained by mid-cycle traders. The timing which reduces the penalty to a minimum is determined by investors' opportunity rate of return and the relationship between the rates of tax and credit on capital gains and losses (Section 6).
- (vi) Increase in the frequency of dividends is likely to benefit stockholders by decreasing the average tax penalty associated with mid-cycle trading (Section 7).

## 2. The Ask Price Path Over the Dividend Cycle

Consider a non-growth stock whose market is dominated by individual investors of a given tax bracket. This stock is expected to generate a year-end dividend perpetuity subject to a personal tax regime similar to our own: dividend tax at an unspecified rate, capital gains tax (CGT) at the effective rate  $t^G$ , and capital loss credit (CLC) at the effective rate  $t^L$ , constrained by  $0 \leq t^L \leq t^G < 1$ ; dividends are

taxed when earned, whereas CGT and CLC are paid and received at the end of the year in which they are realized.<sup>3</sup>

Consistent with these assumptions, the ex-dividend share price obtains the familiar form

$$P_a = \frac{D}{r} \quad (1)$$

where  $D$  represents the post-tax value of dividends and  $r$  investors' post-tax opportunity rate of return on equal-risk investment. The absence of  $t^G$  and  $t^L$  from (1), despite the presence of CGT and CLC, implies the assumption that the market for this stock is dominated by ex-dividend trading, expected to involve neither capital gains nor losses. The analysis of ask and bid price behavior over the dividend cycle, conducted respectively in this section and in Section 3, provisionally adopts this assumption based on the presumption that investors seek to minimize the tax impact attached to a given flow of dividends.

Let  $z$  be the fraction of the cycle over which the share is held by the mid-cycle seller ( $1-z$  is the fraction over which the share is held by the buyer) and let  $P^s$  be the lowest supply price accepted by the seller at any given point in the cycle, conveniently referred to as ask price.

Given an annual opportunity rate of return  $r$ , the seller should earn over a fraction  $z$  of the year the rate

$$(1+r)^z - 1 = \frac{(P^s - P_a) - \frac{(P^s - P_a)t^G}{(1+r)^{1-z}}}{P_a}, \quad 0 \leq z \leq 1 \quad (2)$$

where the first term in the numerator is capital gain and the second term the discounted year-end tax paid on that gain. The ask price is derived by solving this equation for  $P^s$

$$P^s = P_a \frac{1+r-t^G}{(1+r)^{1-z}-t^G} \quad (3)$$

This price is increasing in  $z$ ,  $r$ , and  $t^G$ . Given  $r$  and  $t^G$ , it reaches a minimum of  $P^s = P_a$  at  $z = 0$  where the expression is reduced to (1) and a maximum at  $z = 1$ . Denoting the cum-dividend ask price by  $P_b^s$ , the maximum price  $P^s = P_b^s$  is given by

$$\begin{aligned}
P_b^s &= P_a \frac{1+r-t^G}{(1+r)^0-t^G} \\
&= P_a + P_a \frac{r}{1-t^G} \\
&= \frac{D}{r} + \frac{D}{1-t^G} \quad (4)
\end{aligned}$$

According to (4), an annual price appreciation of  $D/(1-t^G)$  would be necessary in order to allow an investor, buying a share in the beginning of the year for  $P_a$  and selling it at its end for  $P_b^s$ , to earn the post-tax rate  $r$ .

The instantaneous rate of ask-price appreciation  $u^s$  at any  $z$  ( $0 \leq z \leq 1$ ) may be derived from (3)

$$\begin{aligned}
u^s &= \frac{1}{P^s} \cdot \frac{\partial P^s}{\partial z} \\
&= \frac{(1+r)^{1-z} \ln(1+r)}{(1+r)^{1-z} - t^G} \quad (5)
\end{aligned}$$

As expected, this rate is positive and increasing in  $t^G$ . Given  $t^G = 0$ , it is reduced to  $\ln(1+r)$ , the instantaneous equivalent of the annual rate  $r$ . A surprising feature of (5) is the implied dependence of  $u^s$  on  $z$ : the rate of price appreciation accelerates over the dividend cycle, as indicated by the sign of the partial derivative in  $z$

$$\frac{\partial u^s}{\partial z} = \frac{t^G (1+r)^{1-z} \ln^2(1+r)}{[(1+r)^{1-z} - t^G]^2} > 0 \quad (6)$$

As in the absence of CGT, in its presence the rate of price increase is an increasing function of  $r$ .<sup>4</sup>

The range of values assumed by  $u^s$  may be found by substituting  $z = 0$  and  $z = 1$  in (5). The lowest rate of price appreciation prevails ex-dividend at  $z = 0$

$$u^s = \frac{(1+r) \ln(1+r)}{1+r-t^G}$$

and the highest rate at  $z = 1$

$$u^s = \frac{\ln(1+r)}{1-t^G}$$

The last result may be inferred from (4) which implies the year-over rate of price increase

$$\frac{P_b^s - P_a}{P_a} = \frac{r}{1-t^G}.$$

### 3. The Bid Price Path Over the Dividend Cycle

The highest share price offered by the mid-cycle buyer, the bid price, should allow the buyer to earn the post-tax opportunity rate of return  $r$  under the best available resale policy. Hopefully such a policy calls for resale on an ex-dividend date in consonance with the assumed domination by ex-dividend trading. Indeed, benefits are maximized by attracting the highest value of CLC through resale at the lowest point in the cycle. Yet, since the lowest obtainable price ( $D/r$ ) is repeated at the end of every cycle as is the largest obtainable tax credit, the mid-cycle buyer maximizes benefit by reselling the share and realizing the tax credit at the *earliest* opportunity of an ex-dividend price. Therefore, in the following discussion it is assumed that the bid price offered by the mid-cycle buyer allows the buyer to earn the rate  $r$  over a fraction  $1-z$  of the same cycle in which the share is purchased.

Denoting the bid (demand) price by  $P^d$ , the buyer's sub-annual rate of return is

$$\begin{aligned}
(1+r)^{1-z} - 1 &= \\
&= \frac{D - (P^d - P_a) + (P^d - P_a)t^L}{P^d} \\
&= \frac{D - (P^d - P_a)(1-t^L)}{P^d}, \quad 0 \leq z \leq 1 \quad (7)
\end{aligned}$$

where  $(P^d - P_a)$  is the capital loss and  $(P^d - P_a)t^L$  the associated tax credit. The bid price is derived by solving this equation for  $P^d$

$$P^d = \frac{D + P_a(1-t^L)}{(1+r)^{1-z} - t^L} \quad (8)$$

but since  $D = P_a r$ ,

$$P^d = P_a \frac{1+r-t^L}{(1+r)^{1-z} - t^L} \quad (9)$$

This expression resembles the ask price given by (3) with the exception of the credit rate  $t^L$  replacing the tax rate  $t^G$ . It follows that under year-end dividend policy the behavior of the bid should differ from that of the ask only to the extent of the difference between the credit and tax rates. Specifically, given the condition  $t^L < t^G$ , the relationships  $\partial P^s / \partial t^G > 0$  in (3) and  $\partial P^d / \partial t^L > 0$  in (9) would imply  $P^s > P^d$  for any given  $z$  namely, the existence of

a price spread over the period separating dividend dates. This feature is reflected in Fig. 1 where the two price paths are drawn over one dividend cycle. These paths show an identical ex-dividend minimum of  $P^d = P^s = P_a$ , but a different maximum. The bid price maximum  $P^d = P_b^d$  is

$$P_b^d = \frac{D}{r} + \frac{D}{1 - t^L} \quad (10)$$

which, based on the relationship  $t^L \leq t^G$ , is potentially lower than the maximum asked [eq. (4)].

Following (5), the instantaneous rate of bid price appreciation  $u^d$  is given by

$$u^d = \frac{(1+r)^{1-z} \ln(1+r)}{(1+r)^{1-z} - t^L} \quad (11)$$

which is clearly lower than or equal to  $u^s$  for any given  $z$  when  $t^L \leq t^G$ .

Results presented thus far indicate that the effect of capital taxation on the behavior of share prices over the dividend cycle is more complex than commonly perceived. Lack of parity in the treatment of capital gains and losses reflected by the inequality  $t^L < t^G$  would create its own bid-ask price spread which would increase over the cycle, disappearing only on ex-dividend date. According to these results any discussion of the effect of taxes on "the" market price is misleading, since it ignores the asymmetry in the effect of trading on mid-cycle buyers and sellers. One implication of these results is that the price drop on ex-dividend date is not simply  $D/(1 - t^G)$  as argued by Elton and Gruber ([5], eq. 2), but

either  $D/(1 - t^G)$  or  $D/(1 - t^L)$ , which may differ depending upon the relationship between  $t^G$  and  $t^L$ . A broader implication is that under  $t^L < t^G$  sellers and buyers must share a tax penalty when trading at any point in the cycle other than on ex-dividend date. This conclusion rejects a belief shared by Elton and Gruber, according to which existing competition in the capital market automatically ensures the same systematic rate of return for trading throughout the dividend cycle.

#### 4. Why Market Domination by Ex-dividend Trading?

To show that a market dominated by systematic trading at any point in the cycle other than on ex-dividend date would be in disequilibrium, forcing a move toward ex-dividend trading, consider the extreme opposite case of a market dominated by cum-dividend trading.

Under such conditions the prevailing cum-dividend market price  $P_b$  assumes the familiar form of a discounted perpetual stream of post-personal-tax dividends  $D$ , with the first payment to commence immediately

$$P_b = \frac{D}{r} + D$$

Focusing for simplicity on extreme values of the cycle, it is readily seen that the ex-dividend ask price must be

$$P_a^s = \frac{D}{r} + D - \frac{D}{1 - t^L} = \frac{D}{r} - \frac{Dt^L}{1 - t^L}$$

since this is the only price preserving a seller's return at the rate  $r$ . The ex-dividend bid price is

$$P_a^d = \frac{D}{r}$$

since this price allows the buyer to earn exactly the opportunity rate  $r$  under the best available resale policy. This policy calls for ex-dividend resale at the price  $D/r$ , which may take place at the end of any future cycle as indicated by the implied relationship  $D/P_a^d = r$ .

The obtained ex-dividend relationship of  $P_a^d > P_a^s$  is inconsistent with market equilibrium since, as illustrated in Fig. 2, it implies an incentive to trade the stock ex-dividend. But such trading switches the market to an

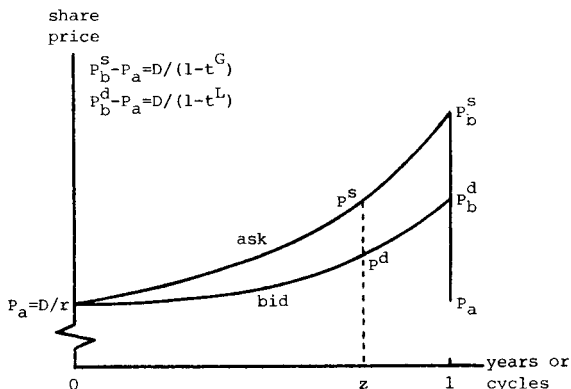


FIGURE 1 Bid and ask price paths over the dividend cycle.

irreversible domination by ex-dividend trading, in the process pushing prices to levels consistent with such domination, along the lines described in Sections 2 and 3, and in Fig. 1.

### 5. Changing the Timing of Annual Distribution

Bid and ask price behavior has been analyzed thus far under the simplifying assumption that dividends are paid at the end of the year. The analysis is now expanded to explore price behavior under a policy of annual distribution at *any* fixed date during the tax year, given that realized CGT and CLC are due, as assumed above, at the end of the year.

With  $f$  denoting the fraction of the year elapsing before the dividend date, Fig. 3 indicates the ask price behavior be analyzed separately for the portion of the cycle coinciding

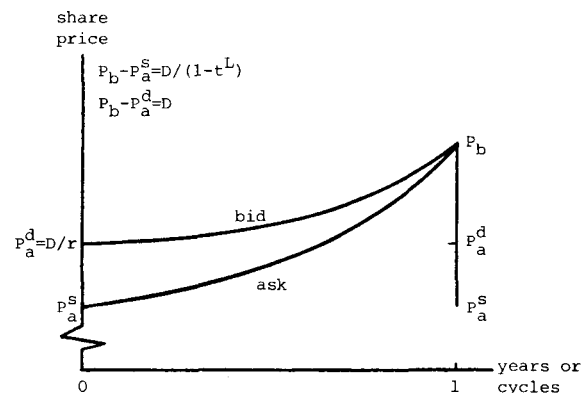


FIGURE 2 Bid and ask price paths over the dividend cycle under hypothetical domination by cum-dividend trading.

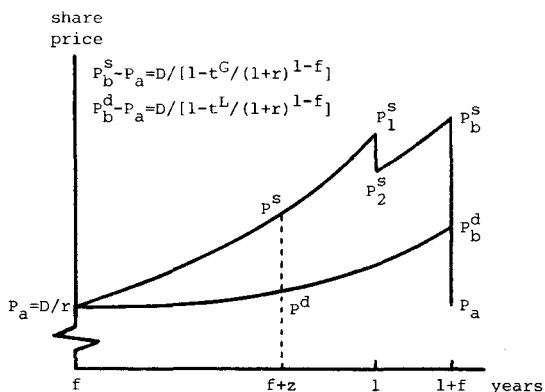


FIGURE 3 Bid and ask price paths over the dividend cycle with mid-cycle dividend.

with that fraction ( $1 \leq f + z \leq 1 + f$ ), and for the remainder of the cycle ( $0 \leq f + z \leq 1$ ). It also indicates that a similar distinction not apply to the bid price.

#### The ask price path

Case I:  $0 \leq f + z \leq 1$

Derivation of the ask price over this portion of the cycle resembles that of equation (3), but the resulting price reflects an increase in the effective CGT rate due to a shorter delay in tax payment for any given  $z$

$$P^S = P_a \frac{1 + r - \frac{t^G}{(1 + r)^{-f}}}{(1 + r)^{1-z} - \frac{t^G}{(1 + r)^{-f}}} \quad (12)$$

$$= P_a \frac{(1 + r)^{1-f} - t^G}{(1 + r)^{1-f-z} - t^G}$$

The fraction  $f$  assumes a value between zero and one, defining the maximum value that can be assumed by the fraction  $z$  within the constraint  $f + z \leq 1$ . Inspection of this price expression for extreme values of  $f$  yields the following. Under beginning-of-the-year dividend  $f = 0$  such that (12) is simplified to (3), a situation requiring no further comment. Under year-end dividend  $f = 1$  and, due to the maximum constraint on the sum of the fractions, the function is reduced to a point  $P^S = P_a$  at  $z = 0$ .

Case II:  $1 \leq f + z \leq 1 + f$

Following (3) and (12), the ask price over this portion of the cycle is

$$P^S = P_a \frac{1 + r - \frac{t^G}{(1 + r)^{1-f}}}{(1 + r)^{1-z} - \frac{t^G}{(1 + r)^{1-f}}} \quad (13)$$

$$= P_a \frac{(1 + r)^{2-f} - t^G}{(1 + r)^{2-f-z} - t^G}$$

showing a decrease in the effective CGT rate resulting from a longer delay in tax payment for any given  $z$ .

Under beginning-of-the-year dividend  $f = 0$  forces  $z = 1$  by the constraint  $f + z \geq 1$ , reducing the function to a point of value

$$P^s = P_a \frac{(1+r)^2 - t^G}{1+r-t^G}$$

Under year-end dividend the value  $f = 1$  reduces (13) to (3), as the value  $f = 0$  reduced (12) to (3) in Case I. The year-end price discontinuity indicated by these equations and illustrated in Fig. 3 results from the assumed discrete annual nature of personal CGT payments. The year-end ask price drop  $P_1^s - P_2^s$  is calculated by subtracting (13) from (12) after substituting  $f + z = 1$  in both equations

$$P_1^s - P_2^s = P_a \frac{(1+r)^{1-f} - t^G}{1-t^G} - P_a \frac{(1+r)^{2-f} - t^G}{1+r-t^G}$$

The price drop due to dividend payment at the end of the cycle  $P_b^s - P_a$  is obtained by substituting  $z = 1$  in (13)

$$P^s = P_b^s = P_a + \frac{D}{1 - \frac{t^G}{(1+r)^{1-f}}}$$

The sought drop in price is given by the second term of this expression.

### The bid price path

Following the derivation of (9), the bid price is given by

$$P^d = P_a \frac{(1+r)^{2-f} - t^L}{(1+r)^{2-f-z} - t^L} \quad (14)$$

an expression resembling the ask price in Case II [eq. (13)], with the exception of the potential difference between tax and credit rates. This similarity indicates that, given the equality  $t^L = t^G$ , the two price paths would converge over the portion of the cycle defined by the constraint  $1 < f + z < 1 + f$ .

Comparison of (14) and (12) shows that the bid and ask price paths would *not* converge over the portion of the cycle constrained by  $0 \leq f + z \leq 1$  where the ask price exceeds the bid for any given  $f$  and  $z$  even when  $t^L = t^G$ . This rather surprising result is explained by the fact that a sale before the end of the year ( $f + z \leq 1$ ) entails a CGT payment at the end of the same year, whereas a purchase during that period entails a CLC receipt only at the end of the following year. In a broader sense, this finding indicates that the phenomenon of

tax-induced price spread and the implied differences between results of this study and those of Elton and Gruber [5] do *not* disappear under a tax regime of  $t^L = t^G$ .

Further examination of equation (14) shows that under beginning-of-the-year dividend ( $f = 0$ ) it is simplified to

$$P^d = P_a \frac{(1+r)^2 - t^L}{(1+r)^{2-z} - t^L} \quad (15)$$

reflecting a decline in the effective CLC rate by comparison to (9). In line with the difference between (14) and (12) in the general case  $0 \leq f \leq 1$ , in the special case  $f = 0$  the bid price given by (15) is below the ask price indicated by (3) for any given  $z$  even if  $t^L = t^G$ .

The ex-dividend ask price drop  $P_b^d - P_a$  is calculated by substituting  $z = 1$  in (14). In view of the equality  $D = P_a r$ , that equation becomes

$$P^d = P_a + \frac{D}{1 - \frac{t^L}{(1+r)^{1-f}}}$$

in which the second term stands for the drop in price.

Unlike the ask price, the bid price path shows no year-end discontinuity—*asymmetry* consistent with the principle that a change in the calendar year matters only to the extent that it causes a change in the year in which the gain or loss is realized. A mid-cycle change in the year of realization may affect the gain of mid-cycle sellers but not the loss of mid-cycle buyers, since the latter are always better off reselling their shares at the end of the current cycle.

## 6. The Dividend Date as a Decision Variable

Results of previous sections indicate that under a tax regime characterized by the inequality  $t^L < t^G$  the market for a stock paying dividends at any fixed date would be dominated by ex-dividend trading. Although this conclusion offers no clue as to the firm's optimum choice of dividend date or the proper frequency of distribution, such clues are contained in results pertaining to the bid and ask price behavior over the dividend cycle. It could be argued that, other things held constant, the value of a stock traded in an environment of

imperfect foresight would be increased by a dividend policy designed to reduce the tax penalty associated with mid-cycle trading, and therefore the tax-induced spread. The analysis in this section is aimed at identifying a best dividend date, subject to the constraint of annual distribution, on the assumption that the firm seeks to bring to a minimum the average spread over the cycle.

In terms of Fig. 3, the objective of minimizing the average spread is interpreted geometrically as an attempt to minimize the area bounded between the ask and bid price paths over a full cycle. This can be accomplished by first integrating the price functions with respect to  $z$  over the interval  $0 \leq z \leq 1$  (treating  $f$  as a parameter), then selecting an  $f$  value which minimizes the difference between the calculated ask and bid integrals. Noting the continuity of (12), (13), and (14) over relevant intervals of  $z$ , the following integrals are evaluated after scaling the three price equations to  $P_a = 1$ .

Ask price, Case I ( $0 \leq f + z \leq 1$ ):

$$\int_0^{1-f} \frac{(1+r)^{1-f} - t^G}{(1+r)^{1-f-z} - t^G} dz$$

$$= -\frac{(1+r)^{1-f} - t^G}{t^G \ln(1+r)} \left[ \ln(1+r)^{1-f} + \ln \frac{1-t^G}{(1+r)^{1-f} - t^G} \right] \quad (16)$$

Ask price, Case II ( $1 \leq f + z \leq 1 + f$ ):

$$\int_{1-f}^1 \frac{(1+r)^{2-f} - t^G}{(1+r)^{2-f-z} - t^G} dz$$

$$= -\frac{(1+r)^{2-f} - t^G}{t^G \ln(1+r)} \left[ \ln(1+r)^f + \ln \frac{(1+r)^{1-f} - t^G}{1+r - t^G} \right] \quad (17)$$

Bid price ( $0 \leq z \leq 1$ ):

$$\int_0^1 \frac{(1+r)^{2-f} - t^L}{(1+r)^{2-f-z} - t^L} dz$$

$$= -\frac{(1+r)^{2-f} - t^L}{t^L \ln(1+r)} \left[ \ln(1+r) + \ln \frac{(1+r)^{1-f} - t^L}{(1+r)^{2-f} - t^L} \right] \quad (18)$$

All logarithmic functions contained in these integrals have positive arguments and so are the integrals themselves. As indicated by Fig. 3 and as can be ascertained by differentiation, of these integrals the first is monotone decreasing in  $f$  while the other two are monotone increasing in  $f$ . The area bounded by the price paths between  $z = 0$  and  $z = 1$  is given by the sum of (16) and (17) minus (18) which, given  $t^L < t^G$ , is partially increasing in  $t^G$  and decreasing in  $t^L$ , but may be increasing or decreasing in  $f$ . In view of its inconvenient mathematical form and the modest degree of precision called for, the value of  $f$  minimizing that area is searched for by substituting increasing values of  $f$  ( $0 \leq f \leq 1$ ) in these integrals.<sup>5</sup>

The effect of the timing of dividends on the average bid-ask spread under feasible values of  $r$ ,  $t^G$ , and  $t^L$  is illustrated by numerical examples displayed in the Table. Calculations show that the optimal  $f$  is a function of the ratio  $t^L/t^G$ , assuming the approximate values  $f = .7$  and  $f = .8$  for  $t^L/t^G$  ratios of .50 and .75, respectively, and the exact value  $f = 1.0$  for  $t^L/t^G = 1.0$ . It is worth noting that optimal mid-year distribution is consistent with the mathematical character of the integrals involved—all three are nonlinear in  $f$ . As implied by the similarity of (17) and (18) and

TABLE  
Penalty Sustained by Mid-cycle Trader as a Fraction of Return to Seller, under Best and Worst Timing of Dividends§

Opportunity Rate of Return	Ratio of CLC Rate, $t^L/t^G$	Rate of CGT, $t^G$		
		.20	.25	.30
.05	.50	.107-.113	.140-.146	.171-.179
	.75	.058-.065	.076-.085	.094-.108
	1.00	0-.013	0-.015	0-.020
.10	.50	.105-.116	.136-.148	.167-.183
	.75	.056-.071	.073-.092	.093-.115
	1.00	0-.021	0-.028	0-.036
.15	.50	.103-.118	.133-.151	.164-.185
	.75	.056-.076	.072-.099	.090-.123
	1.00	0-.030	0-.040	0-.050

§ The best timing, at which point the average spread is at a minimum, is at about  $f = .7$ ,  $f = .8$ , and (exactly)  $f = 1.0$ , for  $t^L/t^G$  ratios of .50, .75, and 1.0, respectively. The worst timing, at which point the average spread is at a maximum, is at  $f = 0$  in all cases.

confirmed by numerical results, the spread can be eliminated only under a tax regime of  $t^L = t^G$ . Results show that the effect of the timing of dividends on the average spread is such that the penalty sustained by mid-cycle traders may increase by a few percentage points due to deviation from optimal timing. The effect of timing on the spread is seen to be small by comparison to the spread itself, suggesting that a decrease in the spread can be more effectively achieved by government policy ensuring the equality  $t^L = t^G$  than by corporate dividend policy designed to minimize the impact of a given inequality  $t^L < t^G$ .

### 7. Dividend Frequency as a Decision Variable

Results to this point have been constrained by the assumption of annual dividend payment. The implications of a higher distribution frequency for price behavior and the penalty associated with mid-cycle trading are explored next under the simplifying assumption of semi-annual dividends paid in the middle and end of every year. The argument is subsequently extended to higher distribution frequencies.

Let  $\bar{r}$  be the semi-annual equivalent of the annual rate of return  $r$ , where  $(1 + \bar{r})^2 = 1 + r$ ; let  $\bar{D}$  be the respective semi-annual dividend, related to the annual dividend by  $\bar{D}/\bar{r} = D/r$ ; and let  $\bar{P}_b^d$  and  $\bar{P}_b^s$  be the mid-year cum-dividend bid and ask prices, respectively. Preserving the assumption that the market is dominated by ex-dividend trading, the resulting price paths are drawn in Fig. 4 and may be derived as follows.

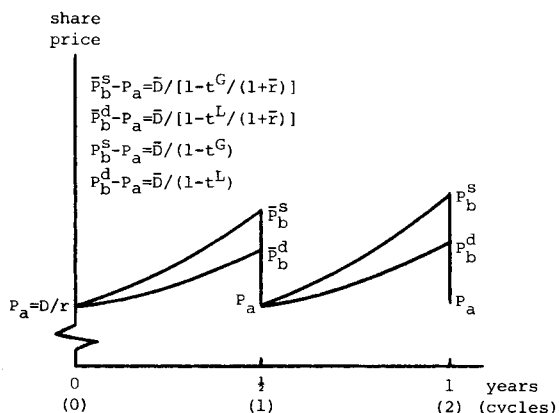


FIGURE 4 Bid and ask price paths over a year with semi-annual dividend.

### Price paths over the first cycle

Whether in the middle or at the end of the year, the ex-dividend price remains the same as under annual dividends

$$P_a = \frac{\bar{D}}{\bar{r}} = \frac{D}{r}$$

again involving no direct impact of CGT or CLC.

Given that CGT and CLC are still paid and received at the end of the year in which they are realized, the bid and ask price paths over the first cycle (first half of the year) remain the same as under annual distribution. While in the case of the ask price the claim of no change is obvious—in the case of the bid price it calls for an explanation. For the claim to be valid it must be shown that a share commands the same bid price during the first six months, regardless of whether dividend and capital loss occur in the middle of the year or at its end. Following (7), a resale at the end of the first cycle should allow the competitive rate of return

$$(1 + r)^{1-z} - 1 =$$

$$\frac{\bar{D} - (P^d - P_a) \left(1 - \frac{t^L}{1 + \bar{r}}\right)}{P^d},$$

$$0 \leq z \leq \frac{1}{2} \quad (19)$$

involving a semi-annual dividend  $\bar{D}$  and a reduced effective CLC rate  $t^L/(1 + \bar{r})$ . Noting the equalities  $\bar{D} = P_a \bar{r}$  and  $(1 + \bar{r})^2 = 1 + r$ , the solution of (19) for  $P^d$  is (9), a solution identical to that obtained under the assumption of year-end sale.

As implied by the original price paths and as indicated by Fig. 4, the mid-year dividend payment involves an ask price drop of

$$\bar{P}_b^s - P_a = \bar{D}/[1 - t^G/(1 + \bar{r})]$$

and a bid price drop of

$$\bar{P}_b^d - P_a = \bar{D}/[1 - t^L/(1 + \bar{r})]$$

### Price paths over the second cycle

The bid and ask price paths over the second cycle (second half of the year) change along the following lines.

Following (2), a share purchased ex-divi-

dend in the beginning of the second cycle should allow a mid-cycle seller the annual rate of return  $r$

$$(1+r)^{z-1} - 1 = \frac{(P^s - P_a) - \frac{(P^s - P_a)t^G}{(1+r)^{1-z}}}{P_a}, \quad \frac{1}{2} \leq z \leq 1 \quad (20)$$

The ask price derived by solving this equation for  $P^s$

$$P^s = P_a \frac{1 + \bar{r} - t^G}{(1+r)^{1-z} - t^G} \quad (21)$$

is lower than that obtained under annual dividend [eq. (3)] for any relevant  $z$ , since  $\bar{r} < r$ . Substitution of  $z = 1$  indicates a reduced ex-dividend price drop of  $P_b^s - P_a = \bar{D}/(1 - t^G)$ .

The rate of return earned on a share purchased during the second cycle and resold at its end is obtained by substituting  $\bar{D}$  for  $D$  in (7)

$$(1+r)^{1-z} - 1 = \frac{\bar{D} - (P^d - P_a)(1 - t^L)}{P^d}, \quad \frac{1}{2} \leq z \leq 1 \quad (22)$$

yielding the bid price

$$P^d = P_a \frac{1 + \bar{r} - t^L}{(1+r)^{1-z} - t^L} \quad (23)$$

in which  $\bar{r}$  takes the place of  $r$  in the numerator of (9). As in the case of the ask price, equation (23) shows that the bid price over the second cycle is lower than that obtained under annual dividend for any admissible  $z$ , since  $\bar{r} < r$ . It also indicates a smaller ex-dividend price drop of  $P_b^d - P_a = \bar{D}/(1 - t^L)$ .

#### *Penalty imposed on mid-year trading*

A switch from a single year-end distribution to semi-annual distribution with dividends paid in the middle and end of the year causes no change in the bid-ask spread over the first six months of every year, but a decrease in the spread for given values of  $z$  over the second six-month period. The latter is apparent from a comparison of the price expressions [(21) vs. (3), (23) vs. (9)], showing that the only change caused by the switch is the substitution of  $r$  in the numerator by the lower rate  $\bar{r}$  due to smaller accumulation of earnings at any given

$z$ . This substitution necessarily involves a decrease in the average price spread since, given  $z$  and the parameters  $t^G$  and  $t^L$ , both prices are monotone decreasing in  $r$ , converging upon the same limit of  $P_a$  under  $r \rightarrow 0$ .

#### *Penalty under higher dividend frequencies*

Conclusions drawn about the effects of increasing the dividend frequency from once to twice a year can now be used to argue that the average price spread and tax penalty on mid-cycle trading are decreased by cutting in half any distribution period and, therefore, are generally decreased by an increase in the frequency of dividends. The argument is as follows: Any cycle not straddling tax years can be sub-divided into two equal-term sub-cycles in the same year. Of these sub-cycles, the first has price paths and spread identical to those of the original cycle up to its mid-point, and the second a spread narrower than that of the original cycle over the parallel period, due to the combination of a smaller accumulation of undistributed earnings at any given point in time and an identical dividend date implying similar tax treatment. Given other factors influencing the firm's dividend schedule, this conclusion suggests that the optimal dividend frequency should increase with  $t^G$  and  $r$  which are directly related to the tax penalty of mid-cycle trading, and decrease with  $t^L$  which is inversely related to that penalty.

## 8. Summary

The taxation of capital gains in common equity investments is likely to have a complex and substantial impact upon share price behavior over the dividend cycle. In the presence of a measurable difference between the effective tax and credit rates of capital gains and losses, this tax should cause an economically significant bid-ask price spread, imposing penalty on stock trading. It is shown that this spread increases over the dividend cycle and temporarily disappears on ex-dividend day; its average size increases with the difference between the tax and credit rates, as well as the market interest rate, but is inversely related to the dividend frequency. It follows that this tax affects the pattern of stock trading, causing transactions to be concentrated in the short

periods following dividend dates. The extent of this phenomenon should depend upon the rates of tax, credit, and market interest, and upon the dividend frequency; given a low frequency, it should also depend upon the timing of dividends within the tax year.

These conclusions suggest the roles played by firms and investors in attempting to avoid the tax penalty associated with stock trading. They also suggest the advantage of an accommodating tax regime: only full parity between the effective rates of tax and credit on capital gains enables firms to pursue a distribution policy which eliminates the tax-induced price spread and does not hinder trading throughout the dividend cycle.

### Notes

1. Not to be confused with the common terms “bid” and “ask” referring to observed market prices, these terms are used here in a narrower sense denoting prices which include transaction tax effects but exclude all other transaction costs. The term “price spread” is used respectively.
2. The distinction between short and long term capital gains and losses is ignored. It is also assumed that a tax regime where the *effective* rate of CLC exceeds that of CGT cannot persist. Consistently, under present law the higher nominal rate of CLC is offset by a limit (\$3,000) on the amount of credit—but not gain—that can be claimed in any given year.
3. This is an approximate description of the present regime wherein the tax on dividends beyond a certain small amount must be paid by the end of the quarter in which the dividend is received. Interpretation of the current law with respect to CGT and CLC is more complex. Unlike dividend income which is subject to tax but never receives credit,

capital gains and losses incurred during the year must be offset against each other to determine the net taxable gain for the entire year.

4. This claim can be proved by restating the condition  $\partial u^s / \partial r > 0$  in terms of an equivalent condition

$$Q = 2[(1+r)^{1-z} - t^G] - [(1+r)^{1-z} + t^G](1-z)\ln(1+r) > 0$$

which is demonstrated for  $0 \leq z \leq 1$  by substituting the minimum value  $r = 0$  after showing that  $\partial Q / \partial r > 0$  for feasible values of  $r$  (say,  $0 \leq r < 1$ ).

5. An obvious alternative procedure is to differentiate the signed area with respect to  $f$ , set the derivative at zero, and solve for the optimal  $f$ . This alternative is not more attractive, however, since the derivative in  $f$  is unamenable to analytical solution and would require a similar approximation of the optimal  $f$ .

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